Appendix A

Diffusion equation for skin effect

As an interesting aside, related to the 'Nanowire' PhD project in our group researching on-chip communication, we note that the response of the skin-effect-only copper channel, and a characteristically terminated on-chip RC-limited wire are the same. In both cases, a diffusion equation can be used to describe the channel [Ramo], [Kerst]. For the skin effect in a cylindrical conductor, the magnetic field is written in the diffusion equation. In an on-chip RC-limited wire, the voltage is written in the diffusion equation (charge diffusion). More detail is given below.

In an RC-limited wire, the diffusion equation is [Dally-3]:

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t}. \tag{1}$$

For the skin effect, the diffusion equation is [Kerst]:

$$\frac{\partial^2 B}{\partial x^2} = \sigma\mu\frac{\partial B}{\partial t}, \tag{2}$$

where B is the magnetic field. For a cylindrical conductor with an axial current, this equation becomes [Kerst]:

$$\frac{\partial B_\theta}{\partial t} = \frac{1}{\mu\sigma}\left(\frac{\partial^2 B_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r^2}\right), \tag{3}$$

with r the direction of the radius and where B_θ is the azimuthal magnetic field.

The diffusion process in the case of the skin effect in a cylindrical conductor is that of the magnetic field into the conductor. At time t=0, the magnetic field is only present on the outer edge of the conductor, whilst at t=∞ the field has penetrated the entire conductor. In the intervening time, a diffusion process takes place in the radial direction.

Appendix B

Jitter during the integration period

Jitter during the integration period T_c will effectively lead to duty cycle distortion, which causes inter-carrier interference. We have to make sure that this effect does not invalidate the analysis in Chapter 3. From [McNeill], the jitter variance as a function of measurement time is:

$$\sigma_t = \kappa\sqrt{\tau}\,, \tag{1}$$

where κ is a time domain figure of merit. When we fill in T_c for τ in Eq. 1 and divide by T_c, we obtain the following formula:

$$\frac{\sigma_t}{T_c} = \frac{\kappa}{\sqrt{T_c}}\,. \tag{2}$$

To provide a state-of-the-art estimation for κ, we use the following formulas from [McNeill]:

$$S_\phi(f) = \frac{N}{f^2}\,, \tag{3}$$

$$\kappa = \frac{\sqrt{N}}{f_0}\,. \tag{4}$$

Using a phase noise of -99.5dBc/Hz at 1MHz offset for the ("state-of-the-art") three-stage ring oscillator in [Eken] we calculate its $\kappa \approx 2\cdot 10^{-9}\sqrt{\text{s}}$. By substituting this value into Eq. 2 and assuming that $T_c > 1\cdot 10^{-10}$s, we can calculate that:

$$\frac{\sigma_t}{T_c} < 2\cdot 10^{-4}\,. \tag{5}$$

In order for the correlation plot to be used for calculating bit error rates in the order of $1\cdot 10^{-12}$, we calculate the 7σ variation to be approximately $1/1000^{th}$ of a period, having little or no effect on carrier orthogonality.

Appendix C

Tables for autocorrelation calculations

As explained in Chapter 5, the autocorrelation function $R_{xx}(\tau)$ is calculated by summing three components $R_0(\tau)$, $R_1(\tau)$ and $R_2(\tau)$ as follows:

$$R_{xx}(\tau) = R_2(\tau + 2T_s) + R_1(\tau + T_s) + R_0(\tau) + R_1(-\tau - T_s) + R_2(-\tau - 2T_s)\,. \tag{1}$$

The tables used for calculating these three components, showing the possible combinations of singlets $v_k(i,j)$ and their probability, are given below. First the table for $R_0(\tau)$ is given, next for $R_1(\tau)$ and finally for $R_2(\tau)$.

The following property of the cross-correlation is used in the calculations:

$$R_{x,-y}(\tau) = R_{-x,y}(\tau) = -R_{x,y}(\tau)\,. \tag{2}$$

A shorthand notation is used when writing down the correlations, e.g. R_{AB} means:

$$R_{AB} = \int_0^{T_s} y_1(t) y_2(t+\tau)\, dt\,, \tag{3}$$

where $y_1(t)$ is the singlet 'A' and $y_2(t)$ is the singlet 'B' (see Table 1 in Chapter 5).

C.1. $R_0(\tau)$

Singlet		***$v_0(i,j)$***	**Probability**	**Term**
A	-1 1 -1	$v_0(1,1)$	1/8	AA
-A	1 -1 1	$v_0(2,1)$	1/8	AA
B	-1 1 1	$v_0(3,1)$	1/8	BB
-B	1 -1 -1	$v_0(4,1)$	1/8	BB
C	1 1 -1	$v_0(5,1)$	1/8	CC
-C	-1 -1 1	$v_0(6,1)$	1/8	CC
D	1 1 1	$v_0(7,1)$	1/8	DD
-D	-1 -1 -1	$v_0(8,1)$	1/8	DD

Table 1. $R_0(\tau)$.

Adding up the terms in the above table, we obtain $R_0(\tau)$:

$$R_0(\tau) = \frac{1}{8}\left(R_{AA} + R_{AA} + R_{BB} + R_{BB} + R_{CC} + R_{CC} + R_{DD} + R_{DD}\right)$$
$$= \frac{1}{4}\left(R_{AA} + R_{BB} + R_{CC} + R_{DD}\right). \tag{4}$$

C.2. $R_1(\tau)$

Singlet 1		Singlet 2		$v_1(i,j)$	Probability		Term
A	-1 **1** **-1**	A	**-1** **1** -1			-	
		-A	**1** **-1** 1	$v_1(1,1)$	$1/8\cdot1/2$	1/16	A·-A
		B	**-1** **1** 1			-	
		-B	**1** **-1** -1	$v_1(1,2)$	$1/8\cdot1/2$	1/16	A·-B
		C	**1** **1** -1			-	
		-C	**-1** **-1** 1			-	
		D	**1** **1** 1			-	
		-D	**-1** **-1** -1			-	
							-1/16(AA+AB)
B	-1 **1** **1**	A	**-1** **1** -1			-	
		-A	**1** **-1** 1			-	
		B	**-1** **1** 1			-	
		-B	**1** **-1** -1			-	
		C	**1** **1** -1	$v_1(2,1)$	$1/8\cdot1/2$	1/16	B·C
		-C	**-1** **-1** 1			-	
		D	**1** **1** 1	$v_1(2,2)$	$1/8\cdot1/2$	1/16	B·D
		-D	**-1** **-1** -1			-	
							1/16(BC+BD)
C	1 **1** **-1**	A	**-1** **1** -1			-	
		-A	**1** **-1** 1	$v_1(3,1)$	$1/8\cdot1/2$	1/16	C·-A
		B	**-1** **1** 1			-	
		-B	**1** **-1** -1	$v_1(3,2)$	$1/8\cdot1/2$	1/16	C·-B
		C	**1** **1** -1			-	
		-C	**-1** **-1** 1			-	
		D	**1** **1** 1			-	
		-D	**-1** **-1** -1			-	
							-1/16(CA+CB)
D	1 **1** **1**	A	**-1** **1** -1			-	
		-A	**1** **-1** 1			-	
		B	**-1** **1** 1			-	
		-B	**1** **-1** -1			-	
		C	**1** **1** -1	$v_1(4,1)$	$1/8\cdot1/2$	1/16	D·C
		-C	**-1** **-1** 1			-	
		D	**1** **1** 1	$v_1(4,2)$	$1/8\cdot1/2$	1/16	D·D
		-D	**-1** **-1** -1			-	
							1/16(DC+DD)

-A	1 **-1 1**	A	**-1 1** -1	$v_1(5,1)$	1/8·1/2	1/16	-A·A
		-A	**1 -1** 1			-	
		B	**-1 1** 1	$v_1(5,2)$	1/8·1/2	1/16	-A·B
		-B	**1 -1** -1			-	
		C	**1 1** -1			-	
		-C	**-1 -1** 1			-	
		D	**1 1** 1			-	
		-D	**-1 -1** -1			-	
							-1/16(AA+AB)
-B	1 **-1 -1**	A	**-1 1** -1			-	
		-A	**1 -1** 1			-	
		B	**-1 1** 1			-	
		-B	**1 -1** -1			-	
		C	**1 1** -1			-	
		-C	**-1 -1** 1	$v_1(6,1)$	1/8·1/2	1/16	-B·-C
		D	**1 1** 1			-	
		-D	**-1 -1** -1	$v_1(6,2)$	1/8·1/2	1/16	-B·-D
							1/16(BC+BD)
-C	-1 **-1 1**	A	**-1 1** -1	$v_1(7,1)$	1/8·1/2	1/16	-C·A
		-A	**1 -1** 1			-	
		B	**-1 1** 1	$v_1(7,2)$	1/8·1/2	1/16	-C·B
		-B	**1 -1** -1			-	
		C	**1 1** -1			-	
		-C	**-1 -1** 1			-	
		D	**1 1** 1			-	
		-D	**-1 -1** -1			-	
							-1/16(CA+CB)
-D	-1 **-1 -1**	A	**-1 1** -1			-	
		-A	**1 -1** 1			-	
		B	**-1 1** 1			-	
		-B	**1 -1** -1			-	
		C	**1 1** -1			-	
		-C	**-1 -1** 1	$v_1(8,1)$	1/8·1/2	1/16	-D·-C
		D	**1 1** 1			-	
		-D	**-1 -1** -1	$v_1(8,2)$	1/8·1/2	1/16	-D·-D
							1/16(DC+DD)

Table 2. $R_1(\tau)$.

Adding up the terms in the above table, we obtain $R_1(\tau)$:

$$R_1(\tau)=\frac{1}{16}\begin{pmatrix}-R_{AA}-R_{AB}+R_{BC}+R_{BD}-R_{CA}-R_{CB}+R_{DC}+R_{DD}\\-R_{AA}-R_{AB}+R_{BC}+R_{BD}-R_{CA}-R_{CB}+R_{DC}+R_{DD}\end{pmatrix}$$

$$=\frac{1}{8}\left(-R_{AA}-R_{AB}+R_{BC}+R_{BD}-R_{CA}-R_{CB}+R_{DC}+R_{DD}\right). \tag{5}$$

C.3. $R_2(\tau)$

Singlet 1		Singlet 2		$v_2(i,j)$	Probability		Term
A	-1 1 **-1**	A	**-1** 1 -1	$v_2(1,1)$	1/8·1/4	1/32	A·A
		-A	**1** -1 1			-	
		B	**-1** 1 1	$v_2(1,2)$	1/8·1/4	1/32	A·B
		-B	**1** -1 -1			-	
		C	**1** 1 -1			-	
		-C	**-1** -1 1	$v_2(1,3)$	1/8·1/4	1/32	A·-C
		D	**1** 1 1			-	
		-D	**-1** -1 -1	$v_2(1,4)$	1/8·1/4	1/32	A·-D
							1/32(AA+AB-AC-AD)
B	-1 1 **1**	A	**-1** 1 -1			-	
		-A	**1** -1 1	$v_2(2,1)$	1/8·1/4	1/32	B·-A
		B	**-1** 1 1			-	
		-B	**1** -1 -1	$v_2(2,2)$	1/8·1/4	1/32	B·-B
		C	**1** 1 -1	$v_2(2,3)$	1/8·1/4	1/32	B·C
		-C	**-1** -1 1			-	
		D	**1** 1 1	$v_2(2,4)$	1/8·1/4	1/32	B·D
		-D	**-1** -1 -1			-	
							1/32(-BA-BB+BC+BD)
C	1 1 **-1**	A	**-1** 1 -1	$v_2(3,1)$	1/8·1/4	1/32	C·A
		-A	**1** -1 1			-	
		B	**-1** 1 1	$v_2(3,2)$	1/8·1/4	1/32	C·B
		-B	**1** -1 -1			-	
		C	**1** 1 -1			-	
		-C	**-1** -1 1	$v_2(3,3)$	1/8·1/4	1/32	C·-C
		D	**1** 1 1			-	
		-D	**-1** -1 -1	$v_2(3,4)$	1/8·1/4	1/32	C·-D
							1/32(CA+CB-CC-CD)
D	1 1 **1**	A	**-1** 1 -1			-	
		-A	**1** -1 1	$v_2(4,1)$	1/8·1/4	1/32	D·-A
		B	**-1** 1 1			-	
		-B	**1** -1 -1	$v_2(4,2)$	1/8·1/4	1/32	D·-B
		C	**1** 1 -1	$v_2(4,3)$	1/8·1/4	1/32	D·C
		-C	**-1** -1 1			-	
		D	**1** 1 1	$v_2(4,4)$	1/8·1/4	1/32	D·D
		-D	**-1** -1 -1			-	
							1/32(-DA-DB+DC+DD)
-A	1 -1 **1**	A	**-1** 1 -1			-	
		-A	**1** -1 1	$v_2(5,1)$	1/8·1/4	1/32	-A·-A
		B	**-1** 1 1			-	
		-B	**1** -1 -1	$v_2(5,2)$	1/8·1/4	1/32	-A·-B

		C	**1** 1 -1	*v₂(5,3)*	1/8·1/4	1/32	-A·C
		-C	**-1** -1 1			-	
		D	**1** 1 1	*v₂(5,4)*	1/8·1/4	1/32	-A·D
		-D	**-1** -1 -1			-	
					1/32(AA+AB-AC-AD)		
-B	1 -1 **-1**	A	**-1** 1 -1	*v₂(6,1)*	1/8·1/4	1/32	-B·A
		-A	**1** -1 1			-	
		B	**-1** 1 1	*v₂(6,2)*	1/8·1/4	1/32	-B·B
		-B	**1** -1 -1			-	
		C	**1** 1 -1			-	
		-C	**-1** -1 1	*v₂(6,3)*	1/8·1/4	1/32	-B·-C
		D	**1** 1 1			-	
		-D	**-1** -1 -1	*v₂(6,4)*	1/8·1/4	1/32	-B·-D
					1/32(-BA-BB+BC+BD)		
-C	-1 -1 **1**	A	**-1** 1 -1			-	
		-A	**1** -1 1	*v₂(7,1)*	1/8·1/4	1/32	-C·-A
		B	**-1** 1 1			-	
		-B	**1** -1 -1	*v₂(7,2)*	1/8·1/4	1/32	-C·-B
		C	**1** 1 -1	*v₂(7,3)*	1/8·1/4	1/32	-C·C
		-C	**-1** -1 1			-	
		D	**1** 1 1	*v₂(7,4)*	1/8·1/4	1/32	-C·D
		-D	**-1** -1 -1			-	
					1/32(CA+CB-CC-CD)		
-D	-1 -1 **-1**	A	**-1** 1 -1	*v₂(8,1)*	1/8·1/4	1/32	-D·A
		-A	**1** -1 1			-	
		B	**-1** 1 1	*v₂(8,2)*	1/8·1/4	1/32	-D·B
		-B	**1** -1 -1			-	
		C	**1** 1 -1			-	
		-C	**-1** -1 1	*v₂(8,3)*	1/8·1/4	1/32	-D·-C
		D	**1** 1 1			-	
		-D	**-1** -1 -1	*v₂(8,4)*	1/8·1/4	1/32	-D·-D
					1/32(-DA-DB+DC+DD)		

Table 3. $R_2(\tau)$.

Adding up the terms in the above table, we obtain $R_2(\tau)$:

$$R_2(\tau)=\frac{1}{32}\begin{pmatrix} R_{AA}+R_{AB}-R_{AC}-R_{AD}-R_{BA}-R_{BB}+R_{BC}+R_{BD} \\ +R_{CA}+R_{CB}-R_{CC}-R_{CD}-R_{DA}-R_{DB}+R_{DC}+R_{DD} \\ +R_{AA}+R_{AB}-R_{AC}-R_{AD}-R_{BA}-R_{BB}+R_{BC}+R_{BD} \\ +R_{CA}+R_{CB}-R_{CC}-R_{CD}-R_{DA}-R_{DB}+R_{DC}+R_{DD} \end{pmatrix}$$

$$=\frac{1}{16}\begin{pmatrix} R_{AA}+R_{AB}-R_{AC}-R_{AD}-R_{BA}-R_{BB}+R_{BC}+R_{BD} \\ +R_{CA}+R_{CB}-R_{CC}-R_{CD}-R_{DA}-R_{DB}+R_{DC}+R_{DD} \end{pmatrix}. \quad (6)$$